Multiscale Quasistatic Homogenization for Laminated Ferromagnetic Cores

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Abstract — **In this paper, we investigate the modeling of ferromagnetic multiscale materials. We propose a computational homogenization method based on the heterogeneous multiscale method (HMM) with inclusion of a hysteresis model. The HMM involves: 1) a macroscale problem that captures the slow variations of the overall solution; 2) many microscale problems that allow to determine the constitutive law at the macroscale. At the microscale, a novel energy consistent hysteresis model is incorporated. As application example, a laminated iron core is considered.**

I. INTRODUCTION

The ferromagnetic cores of electromagnetic devices are often laminated for reducing the losses. In some applications, a precise finite-element analysis of such stacked structures is crucial and homogenization techniques indispensable. Further the nonlinear and irreversible behaviour exhibited by ferromagnetic materials must be accounted for. Phenomenological hysteresis models, e.g. the Preisach and Jiles-Atherton models [1, 2], are commonly used for modeling these materials at the macroscopic level. However, these models (possibly vectorized) lack a true physical background. The micromagnetic models [3] circumvent this drawback by addressing hysteresis at the scale of the Weiss domains and the Bloch walls (nanoscale, see Fig. 1, right). These techniques are definitely relevant, though computationally untractable when dealing with engineering applications. Recently, an energy-consistent hysteresis model has been proposed [4]. It is a thermodynamical approach with a naturally vectorial character. We aim at applying the latter within a computational homogenization method that considers two scales: the macroscale (size of the device) and the microscale (determined by the width of a lamination).

The Heterogeneous Multiscale Method (HMM) [5] is a popular methodology, mainly in mechanical and thermal problems, for studying multiscale materials with a reduced computational cost. It is based on the separation of scales and allows to solve a series of microscale problems for determining a homogenized or average quantity of interest that is directly transferred to the macroscale problem. Lately, it is also gaining interest in electromagnetics—see e.g. [6] and references therein.

A finite element nonlinear multiscale homogenization method was proposed in [6]. As a test case, a laminated core with a reversible material law at the microscale was considered. In this paper, this approach is extended in order to include magnetic hysteresis. We will adapt the multiscale technique developed in [7] for an elasto-plastic material governed by an irreversible constitutive law.

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II. MULTISCALE MODEL

The different scales involved in our model are (Fig. 1.): 1) the *macroscale* for the electromagnetic device at hand; 2) the microscale for the size of a representative volume comprising at least one lamination and one insulation layer; and 3) the nanoscale or *physical* scale at the level of the Weiss domains and the Bloch walls [1, 2]. Only the first two are directly accounted for in our multiscale model. The hysteretic phenomena occurring at the level of the Weiss domains are considered via an energy consistent hysteresis model [4] that is incorporated in the microscale.

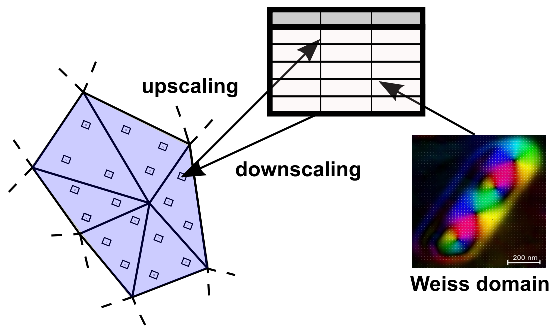

Fig. 1. Different scales found in laminated ferromagnetic cores: from left to right, 1) the macroscale (device); 2) the microscale (lamination + insulation layer); 3) Weiss domains.

We adopt a magnetic vector potential ($\boldsymbol{a}$) formulation, i.e. the weak form of Ampère law, for both the macro and the micro problem. The nonlinear constitutive law $\boldsymbol{h} = \boldsymbol{h}(\boldsymbol{b})$, relating the magnetic field $\boldsymbol{h}$ and the induction $\boldsymbol{b} = \operatorname{curl} \boldsymbol{a}$, comprises irreversible phenomena for the micro problem. Further, we suppose that hysteresis and eddy current losses can be decoupled and we neglect the latter in this abstract to simplify the exposition. A macro problem is defined on a coarse mesh covering the entire domain and many micro problems are defined on small, finely meshed areas around some points of interest of the macro mesh.

Hereafter, vector quantities are in bold. The spatial variation at the macroscale $\boldsymbol{x}$ is related to that of the microscale as $\boldsymbol{y} = \boldsymbol{x}/\varepsilon$. The subscripts M and m refer to quantities at the macroscale and microscale, respectively. The superscript ε refers to quantities with a rapid spatial variation.

A. *Macro problem*

The macro problem is governed by the following weak form: find $\boldsymbol{a}_M(\boldsymbol{x})$ such that

$$\Big(\boldsymbol{h}_M(\boldsymbol{b}_M), \operatorname{curl} \boldsymbol{a}'_M\Big)_{\Omega_M} - \Big(\boldsymbol{j}_s, \boldsymbol{a}'_M\Big)_{\Omega_s} = 0\,, \qquad (1)$$

is verified for all $\boldsymbol{a}'_M(\boldsymbol{x})$ in a suitable function space. The source current density $\boldsymbol{j}_s$ is imposed in Ω_s. The macro constitutive equation $\boldsymbol{h}_M(\boldsymbol{b}_M(\boldsymbol{x}))$ is to be determined (*upscaling*) by solving a micro problem for each Gauss point of the macro mesh, and each micro problem uses $\boldsymbol{a}_M$ for fixing the boundary conditions and the source term (*downscaling*) [6]. We assume that the microstructure contains enough heterogeneities so that the averaging theorem can be applied [7]. The average induction reads:

$$\boldsymbol{b}_M = < \boldsymbol{b}_m^\varepsilon(\boldsymbol{y}) > , \tag{2}$$

what means that $\boldsymbol{b}_M$ is constant in the microdomain Ω_m. The brackets denote an average quantity, i.e. $< \boldsymbol{f} > = \frac{1}{V_m}\int_{\Omega_m} \boldsymbol{f}\, \mathrm{d}\Omega_m$, with V_m the volume of Ω_m.

Once the data from the micro solutions are available, the Newton-Raphson method is applied to tackle (1). Furthermore, the *upscaling* process requires ensuring the Hill-Mandel macro-homogeneity condition, that establishes the equality of macroscopic and (average) microscopic work [7]. The magnetic energy is thus assumed to verify:

$$\boldsymbol{b}_M \cdot \boldsymbol{h}_M = < \boldsymbol{b}_m^\varepsilon(\boldsymbol{y}) \cdot \boldsymbol{h}_m^\varepsilon(\boldsymbol{y}) > . \tag{3}$$

The homogenized magnetic field is then straightforwardly computed by simple averaging $\boldsymbol{h}_M = < \boldsymbol{h}_m^\varepsilon(\boldsymbol{y}) >$ and $\frac{\partial \boldsymbol{h}_M}{\partial \boldsymbol{b}_M}$ by finite differences [6].

B. Micro problem

The weak equation at this level reads:

$$\left(\boldsymbol{h}_m^\varepsilon(\boldsymbol{b}_m^\varepsilon), \operatorname{curl} \boldsymbol{a}_m^{'\varepsilon}\right)_{\Omega_m} = 0\,, \tag{4}$$

where Ω_m is the microdomain. The micro magnetic vector potential $\boldsymbol{a}_m^\varepsilon$ can be expressed in terms of $\boldsymbol{a}_M$, the macro component with slow variations and $\boldsymbol{a}_c^\varepsilon$, the micro unknown that accounts for the rapid variations, i.e. $\boldsymbol{a}_m^\varepsilon(\boldsymbol{y}) = \boldsymbol{a}_M(\boldsymbol{y}) + \boldsymbol{a}_c^\varepsilon(\boldsymbol{y})$.

In order to introduce the hysteretic behaviour, the magnetic flux density $\boldsymbol{b}_m^\varepsilon$ is here split into an empty space component $\boldsymbol{J}_0 = \mu_0 \boldsymbol{h}_m^\varepsilon$ and a material magnetization $\boldsymbol{J}$. The latter is further decomposed into N components subjected to different pinning forces (see [4] for details on the physical background). The partial magnetizations $\boldsymbol{J}_k$ are the internal variables of the hysteresis model :

$$\boldsymbol{h}_m^\varepsilon = \frac{1}{\mu_0}\Big(\boldsymbol{b}_m^\varepsilon - \sum_{k=1}^{N} \boldsymbol{J}_k\Big)\,. \tag{5}$$

Incorporating (5) in (4), together with $\boldsymbol{a}_m^\varepsilon = \boldsymbol{a}_M + \boldsymbol{a}_c^\varepsilon$, the micro problem is fully characterized by: find $\boldsymbol{a}_c^\varepsilon$ such that

$$\left(\operatorname{curl} \boldsymbol{a}_M + \operatorname{curl} \boldsymbol{a}_c^\varepsilon, \operatorname{curl} \boldsymbol{a}_c^{'\varepsilon}\right)_{\Omega_m} - \Big(\sum_{k=1}^{N} \boldsymbol{J}^k, \operatorname{curl} \boldsymbol{a}_c^{'\varepsilon}\Big)_{\Omega_m} = 0\,, \tag{6}$$

is verified for all $\boldsymbol{a}_c^{'\varepsilon}$. The internal variables $\boldsymbol{J}^k$ are initialized to e.g. zero for the first iteration, and then to the converged values from previous computations.

Note that equation (2 a) leads to the periodic boundary conditions for the tangential component of the correction term, i.e. $\int_{\Gamma_m} \boldsymbol{n} \wedge \boldsymbol{a}_c^\varepsilon(\boldsymbol{y}) = 0$.

III. APPLICATION

We consider a laminated ferromagnetic core of a transformer. The multiscale homogenization has been successfully applied to such a stacked core with a nonlinear reversible law, as shown in Fig. 2. The first results concerning the hysteresis model are found in [4]. Ferromagnetic

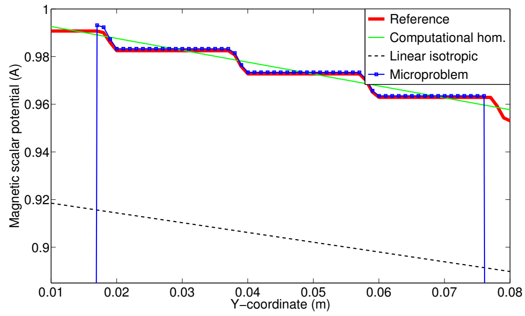

Fig. 2. Cut of magnetic vector potential (z-component) over a 3 laminations - 3 insulation layers microdomain.

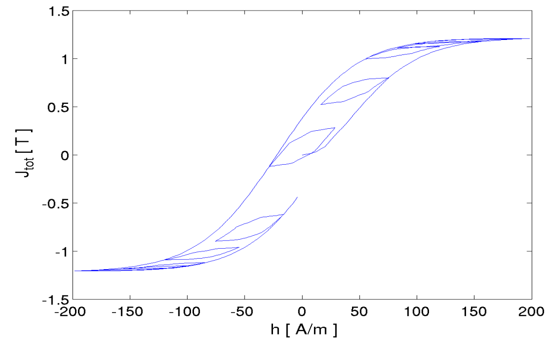

Fig. 3. Main and minor hysteresis loops of M250-50A steel modeled with $\boldsymbol{J}_{1\ldots5}$. The 10-th harmonic is added to the main frequency [4].

materials are well characterized with a reduced number of variables—see hysteresis loops in Fig. 3.

In the extended paper, further details on the theoretical model and results will be given. Results for the coupled implementation will be shown.

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